Eur. J. Mech. B - Fluids 19 (2000) 723–744

S0997-7546(00)00102-3/FLA

Three-dimensional receptivity of boundary layers

Yury S. Kachanov *

Institute of Theoretical and Applied Mechanics, Russian Academy of Sciences, Siberian Division, 630090 Novosibirsk, Russia

(Received 9 April 1999; revised 20 September 1999; accepted 24 September 1999)

Abstract – The paper presents a review of results of some recent (mainly experimental) studies devoted to a quantitative investigation of the problem of receptivity of the 2D and 3D boundary layers with respect to various 3D (in general) external perturbations. The paper concentrates on the mechanisms of excitation and development of stationary and travelling instability modes in a 3D boundary layer on a swept wing, as well as in 2D boundary layers including the Blasius flow and a self-similar boundary layer with an adverse pressure gradient. In particular, the following problems of the boundary-layer receptivity are discussed: (i) receptivity to localized 3D surface roughness, (ii) receptivity to localized 3D surface vibrations, (iii) acoustic receptivity in presence of 3D surface roughness, and (iv) acoustic receptivity in the presence of 3D surface vibrations. All experiments described in the paper were conducted using controlled disturbance conditions with the help of simulation of the stationary and non-stationary perturbations by means of several disturbance generators. This approach gives us the possibility to obtain quantitative results which are independent of any uncontrolled background perturbations of the flow and the experimental model. In contrast to the data obtained at "natural" environmental conditions these results can be directly compared with calculations without any significant assumptions about the physical nature of the disturbances under investigation. The complex (amplitude and phase) coefficients of the boundary-layer receptivity to external perturbations, obtained as functions of the disturbance frequency and the spanwise wavenumber (or the wave propagation angle), represent the main results of the experiments described. These results can be used for the evaluation of the initial amplitudes and phases of the instability modes generated by various external perturbations, as well as for quantitative verification of linear receptivity theories. Several examples of the comparison of experimental results with calculations are also presented in this paper. A brief analysis of the state-of-art in the field is performed and some general properties of different receptivity mechanisms are discussed. © 2000 Éditions scientifiques et médicales Elsevier SAS

laminar boundary layer / three-dimensional receptivity / Tollmien–Schlichting wave / cross-flow wave / cross-flow vortex

1. Introduction

The receptivity of the boundary-layer flows to external perturbations represents a very important aspect of the laminar-turbulent transition problem and has both basic and practical significance. For the first time the receptivity problem has been clearly formulated by Morkovin [1]. Receptivity describes processes by means of which instability waves appear in boundary layers under the action of some external (with respect to the boundary-layer flow) perturbation of various physical sources. These perturbations can enter the boundary layer either through its external edge or from the wall side or through the region of the boundary-layer origin in a vicinity of the body leading edge. Some combined ways of penetration are also possible, for example a scattering of an acoustic wave on a surface non-uniformity.

The first quantitative investigations of the receptivity problem were started in the 1970s. These, in particular, were (i) early experimental papers: Kachanov, Kozlov and Levchenko [2–4]; Shapiro [5]; Kachanov et al. [6]; Aizin and Polyakov [7]; Leehey [8], and others; (ii) theoretical results Mangur [9]; Tam [10]; Kachanov et al. [6] (calculations by Maksimov); see also in Kachanov et al. [4]; Aizin and Polyakov [7]; Murdock [11], and others. For review of some Russian papers cited above see also Leehey [8], Nishioka and Morkovin [12].

* E-mail: kachanov@itam.nsc.ru

In particular, the mechanisms of the leading edge receptivity under the action of the free-stream vortices, the acoustic waves, or the leading-edge vibrations were investigated in these papers. Some mechanisms of the acoustic receptivity not connected with the leading edge were also studied, such as scattering of the acoustic wave on a surface roughness element or on a natural spatial non-uniformity of the boundary layer. The state-of-art in the receptivity problem at that time was also discussed in reviews (Loehrke, Morkovin and Fejer [13]; Reshotko [14]; and Morkovin [15]). The results obtained in this field in the 1970s and beginning of the 1980s were summarized in Kachanov et al. [4] and Nishioka and Morkovin [12]. Later, the receptivity problem was investigated in a great number of theoretical and experimental studies, e.g., Zhigulyov and Tumin [16]; Goldstein and Hultgren [17]; Kerschen [18]; Kerschen, Choudhari and Heinrich [19]; Kozlov and Ryzhov [20]; Morkovin and Reshotko [21]; Choudhari and Streett [22]; Crouch [23]; Choudhari [24]; Dietz [25]; and others. Some results obtained recently, and devoted to the receptivity mechanisms related to the topic of the present paper are briefly discussed below.

From the modern viewpoint the main goal of the analysis of the linear mechanisms of the boundary-layer receptivity is reduced to determining the “initial” amplitudes and phases of the instability modes excited by different external perturbations by means of obtaining the so-called receptivity coefficients or the receptivity functions (see sections 3 and 5 below). Note, however, that strict definitions of the receptivity coefficients can be introduced only in cases when the receptivity mechanisms are localized in the streamwise direction.

The excited instability modes can be amplified (or attenuated) downstream according to the laws described by the linear theory of the hydrodynamic stability. Then, the disturbances (upon reaching high enough amplitudes) enter the nonlinear region of the laminar-turbulent transition. Sometimes the stages of linear excitation and development might be either absent or a transient (short) one. This usually happens when the external perturbations are very intensive and the excited waves have very high initial amplitudes. In such cases the transition process is usually called the ‘bypass transition’. Investigations of the bypass transition scenarios represent a much more complicated problem which is not discussed in the present paper. We concentrate here, in contrast, on the cases of relatively low levels of the external perturbations which are very typical for many practical applications.

From the practical viewpoint two main kinds of the boundary layer instability are the most important, namely: (i) the Tollmien–Schlichting (TS) instability and (ii) the cross-flow (CF) instability. The former is more significant for 2D boundary layers, while the latter dominates for 3D ones. These instability modes can be excited either by free-stream perturbations (stationary or non-stationary) or by various surface non-uniformities. The latter also can be either stationary or non-stationary. In the present paper we concentrate on the following ‘three-dimensional’ receptivity mechanisms: (i) the receptivity to localized surface non-uniformities, (ii) the receptivity to localized surface vibrations, (iii) the acoustic receptivity in the presence of a localized surface roughness, and (iv) the acoustic receptivity in presence of localized surface vibrations. By the time of the beginning of the experimental and theoretical investigations discussed in the present paper, some of these receptivity mechanisms have been studied at length, while others have been rather poorly studied (especially those related to 3D perturbations).

2. Brief review of previous studies in the field

2.1. Excitation of 2D and 3D TS-waves by surface perturbations in 2D boundary layers

This kind of excitation was used in many experiments for the investigation of the linear and nonlinear stability problems, starting with the classic paper, Schubauer and Skramstad [26]. However, the mechanisms of generation of 2D TS-waves in the flat-plate boundary layer by means of unsteady surface non-uniformities were not studied until 1965 theoretically (see Gaster [27]) and until the 1980s experimentally (see Gilyov and Kozlov [28]). Later these receptivity mechanisms were investigated in quite some detail, especially theoretically. In particular, it was found that the disturbance sources localized on the surface excite the TS-waves rather effectively. Probably first theoretical studies of generation of the TS-waves on localized 2D surface vibrations were performed by Terent'ev [29] and Tumin and Fyodorov [30,31]. A similar mechanism was studied for the first time experimentally in Gilyov and Kozlov [28]. The results obtained in this paper were compared with calculations (Terent'ev [32]); Fyodorov [33] studied theoretically an excitation of the instability waves by means of an arbitrary surface perturbation periodical in time; in particular, a vibration, blowing-suction, and heating.

The 3D receptivity mechanisms related to the surface perturbations in 2D boundary layers have been much less investigated. There are a very restricted number of such papers (see, e.g., theoretical works Michalke [34, 35], Michalke and Neemann [36]). A combined experimental and theoretical study of several receptivity problems, including the 3D ones attributed to the surface disturbances, was carried out in Kozlov and Ryzhov [20] where a good qualitative agreement of some experimental results with the asymptotic receptivity theory was found. The problem of excitation of the 3D TS-waves in the 2D boundary layers by surface vibrations had not been studied at all at the time of the beginning of the investigations described below in sections 3.1 and 3.2.

The interest of investigators in the 3D instability waves was rather restricted in the case of 2D subsonic flows due to, probably, the well known Squire theorem (Squire [37]) which is usually interpreted in such a way that the 2D TS-waves are the most dangerous from the viewpoint of transition. However, this interpretation is not quite correct, especially for spatially growing perturbations and in not strictly parallel flows. It was shown recently in (Kachanov and Obolentseva [38,39]) that at certain flow and disturbance parameters the Blasius boundary layer can be more unstable with respect to the 3D TS-waves, propagated at 35 to 40 degrees to the flow direction. This result agrees qualitatively with PSE calculations in (Bertolotti [40]).

It is well known that the 3D waves play also a very significant role at nonlinear stages of transition in the 2D subsonic boundary layers. Thus an investigation of the receptivity of these flows to 3D perturbations is very important (see sections 3.1, 3.2, and 5.2 below).

2.2. Excitation of 2D and 3D TS-waves by acoustics in 2D boundary layers

The mechanisms of generation of the instability waves in 2D boundary layers by means of the acoustic perturbations in the presence of a surface non-uniformity (a roughness) are studied in detail for 2D disturbances. A great number of theoretical papers in this field is discussed in Zhigulyov and Tumin [16]; Goldstein and Hultgren [17]; Kerschen [18]; Kozlov and Ryzhov [20]; Morkovin and Reshotko [21]; Choudhari and Streett [22]; Crouch [23]; Choudhari [24]; and others. It is necessary to note some important experimental papers devoted to this problem. An excitation of the 2D instability waves by acoustics on 2D roughness elements was studied experimentally in Aizin and Polyakov [7]; Kosorygin, Levchenko and Polyakov [41]; Kosorygin [42]; Saric, Hoos and Kohama [43]; Wiegel and Wlezien [44]; Zhou, Liu and Blackwelder [45]; Kosorygin, Radeztsky and Saric [46]) and in other experiments. It was found that acoustics excite rather effectively the 2D TS-waves, even on a microscopically small non-uniformity. The receptivity coefficients were estimated in these studies for different acoustic frequencies, roughness shapes, and acoustic-wave inclination angles (see Zhou et al. [45]). A good agreement between theory and experiment was observed.

Probably first theoretical results on generation of TS-waves by acoustics on a 3D surface roughness element (or a localized 3D suction) were obtained in (Choudhari and Kerschen [47]). Somewhat later this problem was investigated in (Tadjfar and Bodonyi [48]) with the help of non-stationary linearized 3D equations for the asymptotic triple-deck model of the 2D boundary layer. A qualitative comparison of these results with some previous experiments was also performed by Tadjfar and Bodonyi [48]), in particular using the experiments of Gilyov and Kozlov [28] and Tadjfar [49]. The excitation of the TS-waves by acoustics on 3D roughness elements, such as an oblique surface roughness strip and a circular roughness, was investigated experimentally in Zhou et al. [45]. These results were found to be in a good qualitative agreement with theory (Choudhari and Kerschen [47]).

Of course, the most reliable data about the receptivity mechanism can be obtained only with a direct quantitative comparison of experimental and theoretical results, especially the receptivity coefficients. For the 3D acoustic receptivity of the 2D boundary layers such results were not obtained in previous investigations. Recently they were obtained in a study described in section 5.2.

2.3. Excitation of CF-vortices by surface perturbations in swept-wing boundary layer

In the swept-wing boundary layers the TS-waves can also play a significant role in the process of the laminar-turbulent transition. Some mechanisms of generation of the TS-waves in a swept-wing flow by means of surface vibrations were studied theoretically, probably for the first time, in Zhigulyov [50] and Tumin and Zhigulyov [51]. However, the most important mechanism responsible for the 3D boundary-layer transition is associated with the cross-flow instability (see, e.g., Michel et al. [52]; Dallmann and Bieler [53]; Reed and Saric [54]; Kachanov, Tararykin and Fyodorov [55]; Bippes [56]; Kachanov and Tararykin [57]; Kohama, Ohta and Segawa [58]; Gaponenko, Ivanov and Kachanov [59,60]; Kachanov [61]; and others). The CF-instability modes (vortices and waves) are usually dominant on the swept wings because the TS-instability is suppressed initially by a favorable pressure gradient in the vicinity of the wing leading edge.

Until recently quantitative experimental investigations of the mechanisms of generation of the CF-vortices were rather restricted. There are two main ways of causing excitation of the CF-vortices: (i) surface non-uniformities (roughness, waviness, steps etc.) and (ii) steady free-stream perturbations (i.e. some spanwise non-uniformity of the potential flow). The latter way of excitation has been very poorly studied at present, while the former one is investigated more fully.

The significant role of the surface roughness has been well known for many years, starting with early experiments devoted to the swept-wing boundary layer transition (see, for a review, Reed and Saric [54]). However, the first attempt to obtain the linear receptivity coefficients for this problem made in Kachanov and Tararykin [57] was unsuccessful. Their experiment was carried out at controlled disturbance conditions by pasting special small roughness elements onto the surface of a swept-wing model. The experiment showed a quadratic non-linearity of the receptivity mechanism even at the smallest heights of the roughness which excited still measurable (in a vicinity of the roughness) instability modes. The main problem of such experiments is connected with the fact that all modern low-turbulence wind-tunnels are actually not low-turbulence ones in the frequency range below 1 or 0.1 Hz. Despite these difficulties, some significant experimental results on the roughness receptivity problem were obtained at the beginning of 1990s, reported in Radeztsky et al. [62]) who showed that the excitation of the CF-vortices is strongly influenced by a microscopic surface roughness positioned in the vicinity of the swept-wing leading edge. However, quantitative values of the linear receptivity coefficients were also not obtained in this study.

A first detailed theoretical study of the excitation of the CF-instability modes by means of various localized surface perturbations was performed for a swept-wing boundary layer in Fyodorov [63]. Similar problems were investigated in Manuilovich [64]; Crouch [23,65]; Choudhari [24] and Bertolotti [66] with different theoretical approaches. Bertolotti [66] was probably first to perform a quantitative comparison of theoretical and experimental (Deyhle and Bippes [67]) values of the CF-vortex amplitudes generated by roughness elements.

To perform this he used both the receptivity theory and the stability theory. The amplitudes were found to be rather similar.

The first quantitative experimental results on the linear roughness receptivity coefficients were obtained recently in our group and are compared with the calculations by Crouch in section 4.

2.4. Excitation of CF-waves by surface perturbations in swept-wing boundary layer

This problem was studied theoretically in Fyodorov [63] and Bertolotti [66]. Its experimental investigation was started in Ivanov [68] and Ivanov and Kachanov [69,70]. The experimental results were obtained with the help of a new disturbance source designed and tested in our Novosibirsk group in 1991–92 for the excitation of the cross-flow instability waves by means of localized surface vibrations. Similar vibrators were used later very successfully in a large number of experimental studies performed both in our and other research groups. The results were obtained in 2D and 3D boundary layers and were devoted to both stability and receptivity problems.

In particular, it was shown in the papers quoted above that the surface vibrations can generate the CF-instability waves rather effectively. However, the vibration receptivity coefficients were not obtained experimentally until 1994. A significant recent progress in this field is described in section 3.3. Other mechanisms related to the excitation of the CF-waves by various surface perturbations are still rarely studied in a quantitative way, especially experimentally.

2.5. Excitation of CF-waves by acoustics in swept-wing boundary layer

Similarly to two-dimensional boundary layers, the acoustic fields seem to represent one of the possible sources of the CF-instability waves in swept-wing boundary layers. As was shown by Crouch [23,65] the acoustic receptivity mechanism does exist and can play a certain role in the transition process at high enough levels of the acoustic excitation. In these theoretical works a scattering of the acoustic wave on a steady localized surface non-uniformity (a roughness) was investigated. A similar mechanism seems to be possible when the surface non-uniformity is unsteady, i.e. when it represents a localized surface vibrator.

However, the first experimental attempts to detect the acoustic receptivity mechanism in the swept-wing boundary layers were unsuccessful. No influence of the acoustic field on the swept-wing boundary-layer transition and no evidence of generation of the CF-waves by acoustics were found in Müller and Bippes [71] and Takagi et al. [72]. These results were, in a certain way, in contradiction with calculations in Crouch [65] where, in particular, the acoustic receptivity coefficients were obtained for the first time for the excitation of the CF-waves by means of the scattering of the acoustic wave on a localized surface roughness.

A solution of this problem was found in the experiments described below in section 5.1.

3. Three-dimensional vibration receptivity

3.1. Blasius boundary layer

The first experimental quantitative investigation of the 2D boundary-layer receptivity to 3D surface vibrations was performed by Ivanov, Kachanov and Obolentseva [73]. Somewhat later these experimental results were compared directly with calculations by A. Michalke, performed for the experimental conditions of Ivanov et al. [74].

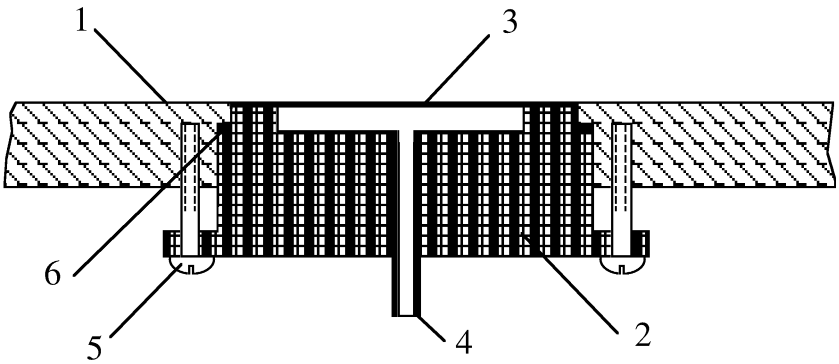

Figure 1. Sketch of circular surface vibrator used in several experiments. 1 — plate surface; 2 — source body; 3 — plastic membrane; 4 — pipe connected to loudspeaker; 5 — adjusting screws; 6 — rubber washer.

The experimental part of the study (as well as the other experiments described below) was conducted at the low-turbulence wind tunnel of the ITAM (Novosibirsk). The free-stream velocity was $U_0 = 9.05$ m/s and the turbulence level was less than 0.02% (measured at frequencies higher than 1 Hz). The boundary layer under investigation evolved on a flat plate mounted under a zero attack angle. The experiments were performed at controlled conditions. The wave-trains of instability waves (harmonic in time and localized in the spanwise direction) were excited in the laminar boundary layer by means of a circular surface vibrator (20 mm in diameter) mounted flush with the wall (*figure 1*). The shape of oscillation of the vibrator membrane was measured accurately by means of two different methods. Typical amplitudes of the vibrations were about $40-50$ microns. Note, that the surface vibrator shown in *figure 1* has been used very effectively in several other experiments devoted to both stability and receptivity problems, in particular in the experiments described below in sections 3.2, 4, and 5.1.1.

All main measurements were conducted by means of a hot-wire anemometer, Ivanov et al. [73,74]. The basic mean flow in these experiments was shown to be in a very good agreement with the Blasius boundary layer (see also Kachanov and Michalke [75] for some more detail characteristics of the basic flow). The local Reynolds number $\mathrm{Re} = U_0\delta_1/\nu$ was 730 at the position of the vibrator, where δ_1 is the boundary layer displacement thickness and ν is the kinematic viscosity of the air. The results were obtained for three different frequencies of vibrations $f = 55.0,\ 81.4$, and 109.1 Hz which corresponded to the non-dimensional frequency parameters $F = 2\pi f\nu/U_0^2 \cdot 10^6 = 64.1$, 94.9, and 127.2 respectively (where ν is the kinematic viscosity of the air).

The theoretical part of the study was performed for the Blasius boundary layer on a basis of a linear receptivity theory developed in Michalke and Al-Maaitan [76] and Michalke [35]. The theory was based on the parallel basic flow assumption and used Fourier analysis and residue theory. In the theory and experiment the same definition of the receptivity function was used.

For a fixed frequency of vibration the 'complex vibration receptivity function'

$$G_{\mathrm{vc}}(\beta) = G_{\mathrm{v}}(\beta)\exp\left[i\varphi_{\mathrm{v}}(\beta)\right] \tag{1}$$

was defined in Ivanov et al. [73,74] as a ratio

$$G_{\mathrm{vc}}(\beta) \stackrel{\mathrm{df}}{=} \frac{B_{\mathrm{inc}}(\beta)}{\tilde{C}_{\mathrm{vc}}(\beta)}, \tag{2}$$

where β is the spanwise wavenumber and

$$B_{\mathrm{inc}}(\beta) = B_{\mathrm{in}}(\beta)\exp\left[i\phi_{\mathrm{in}}(\beta)\right] \tag{3}$$

is the complex initial spectrum (Fourier-integral) of the instability waves excited by the vibrations and determined at the position of the surface vibrator, and

$$\tilde{C}_{\mathrm{vc}}(\beta) = \tilde{C}_{\mathrm{v}}(\beta)\exp\left[i\tilde{\lambda}_{\mathrm{v}}(\beta)\right] \tag{4}$$

is the complex spectrum (double Fourier-integral) of the shape of the surface vibrator determined for so-called resonant modes of this spectrum, which corresponds to the dispersion functions for the 3D TS-waves $\alpha_r = \alpha_r(\beta)$ for every fixed disturbance frequency (where α_r is the streamwise wavenumber of the instability wave). The dispersion functions were obtained both in the theory and experiment. The definition (2) of the receptivity function means that for the receptivity amplitude and phase we have respectively

$$G_{\mathrm{v}}(\beta) = \frac{B_{\mathrm{in}}(\beta)}{\tilde{C}_{\mathrm{v}}(\beta)} \quad \text{and} \quad \varphi_{\mathrm{v}}(\beta) = \phi_{\mathrm{in}}(\beta) - \tilde{\lambda}_{\mathrm{v}}(\beta). \tag{5}$$

All dimensional values were normalized by the potential flow velocity and the boundary-layer displacement thickness $\delta_{1\mathrm{s}}$ at the position of the vibrator.

According to the given definition, the complex receptivity functions were determined both theoretically and experimentally for all three studied frequencies. The amplitude and phase parts of these functions found in experiments (Ivanov et al. [73,74]) are plotted versus the non-dimensional spanwise wavenumber $\beta\delta_{1\mathrm{s}}$ in *figures 2*(*a*),(*b*) respectively. The corresponding theoretical functions are shown in *figures 2*(*c*),(*d*).

First of all, it is seen that the experimental and theoretical results are in a very good qualitative agreement with each other, including a behavior of the receptivity functions with both the frequency and the spanwise wavenumber (or the wave propagation angle). The smallest receptivity amplitudes (*figures 2*(*a*) and *2*(*c*)) are observed for the 2D waves with $\beta\delta_{1\mathrm{s}} = 0$. Then the receptivity amplitudes grow with the spanwise wavenumber. Maximum values of the receptivity amplitudes are observed for $\beta\delta_{1\mathrm{s}} > 0.25$, at least for 3D waves with propagation angels greater then 45 to 50°. The receptivity amplitudes also increase significantly with the vibration frequency for all fixed values of the spanwise wavenumber (or the wave propagation angle). The frequency dependence is much stronger for the 3D perturbations than for the 2D ones.

A quantitative comparison of the receptivity coefficient amplitudes has shown that the theory yields somewhat higher values of the receptivity amplitudes which are greater, in averaged, than the experimental ones by a factor of $\sqrt{2}$. After multiplying the experimental coefficients (*figure 2*(*a*)) by this factor, the receptivity amplitudes become very close to each other for all values of the spanwise wavenumber. Ivanov et al. [74] presume that the difference $\sqrt{2}$ appears due to a different normalization of one of many quantities used in a procedure of determining the receptivity coefficients. Unfortunately due to death of Professor A. Michalke this question remained un-answered.

The experimental and theoretical receptivity phases (*figures 2*(*b*) and *2*(*d*)) have very similar behavior with β and f and display a quite good quantitative agreement with each other (especially for the frequency 81.4 Hz). Some distortions of the experimental phase curves in *figure 2*(*b*) are mainly explained by an experimental error conditioned by an uncertainty of the procedure of upstream extrapolation of the experimental data for the phases growing rapidly downstream. The receptivity phases are seen to be rather weakly dependent on the spanwise wavenumber and, especially, on the disturbance frequency.

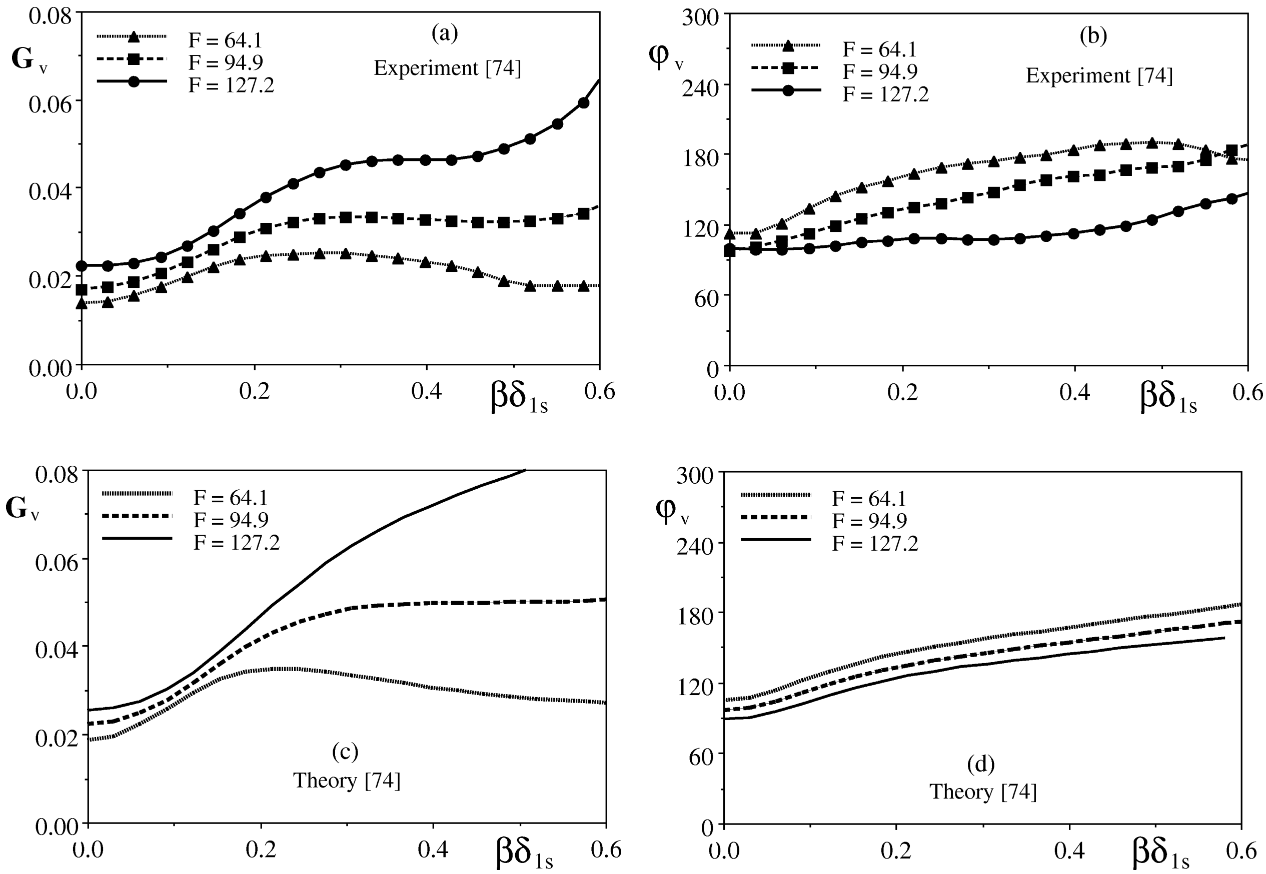

Figure 2. Experimental (*a,b*) and theoretical (*c,d*) amplitudes (*a,c*) and phases (*b,d*) of receptivity functions for vibration receptivity of the Blasius boundary layer (Ivanov et al. [74]).

3.2. 2D boundary layer with adverse pressure gradient

An influence of an adverse pressure gradient (APG) on the 3D vibration receptivity coefficients has been studied experimentally very recently in Kachanov, Koptsev and Smorodsky [77]. The measurements were performed in a two-dimensional self-similar APG boundary layer with a constant Hartree parameter at the free-stream speed $U_0 \approx 9$ m/s and a turbulence level less than 0.02%. The APG was induced over a flat plate (equipped with a flap) by means of an adjustable wall bump mounted on the test-section ceiling above the plate. The measurements were carried out by means of a hot-wire anemometer. The shape of the wall bump and the position of the flap provided a two-dimensional self-similar flow with the Hartree parameter $\beta_{\mathrm{H}} = -0.115$ constant in a wide range of the streamwise coordinate. The experimental procedure was quite similar to that used in the experiments on the Blasius flow (see section 3.1). In particular the measurements were carried out under controlled disturbance conditions. However, the procedure for determining the initial instability wave spectra was different. The 3D instability waves were excited by means of two similar disturbance sources. The first source simulated 3D surface vibrations and served for the main receptivity measurements, while the second one (mounted upstream of the first source) was used for obtaining the streamwise distributions of the perturbations necessary for extrapolation of the amplitudes and phases of the 3D instability waves produced by the first source to its center. Four disturbance frequencies: $f = 55.1$, 81.4, 109.1, and 139.5 Hz were studied.

These frequencies correspond to disturbances positioned inside the neutral stability curve calculated for the 2D waves.

The results of measurements and the data processing (including procedures for a complete Fourier decomposition of the TS-wave trains and surface vibrations into normal oblique modes) gave the possibility of obtaining all the main vibration-receptivity characteristics of the APG self-similar boundary layer with respect to the 3D perturbations. The complex vibration-receptivity coefficients were obtained as functions of the disturbance frequency and the spanwise wavenumber (or the wave propagation angle).

It is found in Kachanov et al. [77] that the typical behavior of the receptivity coefficients with the spanwise wavenumber is very similar in the APG boundary layer to that observed in the Blasius flow in experiments (Ivanov et al. [73,74]) and in theory by Michalke (see Ivanov et al. [74]). Similarly to the Blasius boundary layer, the APG flow is significantly more receptive to the 3D surface vibrations as compared to 2D ones. The receptivity coefficients also grow with the disturbance frequency. The rate of this growth is rather small for the 2D vibrations and increases very rapidly with the spanwise wavenumber (or the wave propagation angle). However, the values of the receptivity coefficients in the APG flow were found to be several times lower than those obtained for the zero streamwise pressure gradient. This rapid reduction of the vibration receptivity coefficients is observed for both 2D and 3D modes and seems to be stronger for the three-dimensional perturbations.

These results are in a very good qualitative agreement with calculations in Michalke and Neemann [36] for 2D modes and in Neemann [78] for 3D modes. During comparison of these data it is necessary to take into account two circumstances. Firstly, in these theoretical works the surface perturbations represent localized blowing-suction disturbances rather than surface vibrations. Secondly, a somewhat different definition of the receptivity coefficients was used in Neemann [78]. In this definition, the spectrum of the excited TS-wave was divided by the spectrum of the normal-to-wall velocity disturbance rather than the wall displacement as in the experiments (see formulas (2) and (4) in section 3.1). For the normal (harmonic in time and space) perturbations the latter definition differs from the definition (2) by dividing the receptivity coefficient by the disturbance frequency. Therefore, the receptivity coefficients used in Neemann [78] tend to infinity when the frequency tends to zero, while the coefficients (2) remain finite for almost all modes. The latter circumstance was essentially used in 3D swept-wing boundary layer for extrapolation of the receptivity coefficients to the zero frequency (see section 4 below).

The behavior of the receptivity phases, studied experimentally in Kachanov et al. [77] in the APG boundary layer, turned out to be almost exactly the same as in the Blasius flow. No any significant distinctions were found. The vibration receptivity phase increases weakly with frequency and wave propagation angle. In contrast to the receptivity amplitudes the values of the phase shifts between the TS-modes and the surface vibration were found to be also almost exactly the same as in the zero pressure gradient case.

The theoretical parametric study in Neemann [78] has shown that the Reynolds number influences rather weakly the surface receptivity coefficients at all studied values of the streamwise pressure gradient, disturbance frequencies, and spanwise wavenumbers.

3.3. Swept-wing boundary layer

The problem of excitation of the CF-instability modes in the swept-wing boundary layer by means of surface vibrations was studied in detail experimentally in Gaponenko, Ivanov and Kachanov [79] and Kachanov, Gaponenko and Ivanov [80]. In particular, the complex coefficients of the vibration receptivity were obtained in these papers as functions of the disturbance frequency and the spanwise wavenumber. A quantitative comparison of these (and other) experimental receptivity characteristics with the theoretical ones, obtained

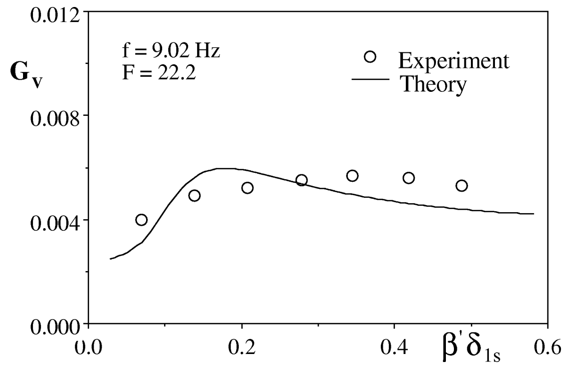

Figure 3. Experimental and theoretical amplitudes of vibration receptivity functions for a swept-wing boundary layer (Crouch et al. [81,82]).

for the experimental conditions and based on a linear receptivity theory (Crouch [23,65]), was performed in Crouch et al. [81,82].

These experiments were carried out on a model of a swept wing with a sweep angle $\chi = 25.0°$. This model was described in detail in Kachanov, Tararykin and Fyodorov [83]; Kachanov and Tararykin [57] and Gaponenko et al. [59,60,79] and used in many Novosibirsk's experiments. The mean flow was produced over a swept-plate by a pressure gradient induced by means of a wall bump positioned over the plate on the test-section ceiling. The structure of the basic flow on the experimental model was studied in detail in Kachanov and Tararykin [57] and Kachanov et al. [83] with the help of a hot-wire anemometer, including measurements with the X- and V-shaped double-wire probes. It was shown that in the region of main measurements the basic flow is adequate locally to a boundary layer on a real swept wing. The surface vibrations were simulated by means of a vibrator similar to that described above in section 3.1. The shape of the vibrations was accurately measured and used during the data processing. This shape was also used in the calculations during comparison of the initial spectra of the instability waves excited in the boundary layer. In the region of the main measurements the potential flow velocity U_0 increased downstream and reached 6.26 m/s at the source position. The local Reynolds number over the vibrator was Re = 498. The measurements were performed at 6 different disturbance frequencies: $f = 24.80$, 21.43, 17.91, 15.00, 12.00 and 9.02 Hz. At the position of the source these frequencies correspond to the non-dimensional frequency parameters $F = 60.7$, 52.4, 43.6, 36.5, 29.5, and 22.2.

The calculations were performed by Crouch et al. [81,82] for the Falkner–Skan–Cooke boundary layer with the quasi-parallel flow approximation within the framework of the linear receptivity theory (see for more detail Crouch [23,65]). The surface perturbations were modeled by linearized boundary conditions derived from a Taylor expansion in terms of the amplitude of vibrations.

The magnitude of the vibration receptivity function obtained in the theory and experiment for frequency of vibration $f = 9.02$ Hz is shown in *figure 3*. A definition of this function was similar to that described in section 3.1, but all values and functions were determined in the (x', z')-coordinate system coupled with the wing chord and span. The experimental and theoretical values are seen to be quite close to each other. The corresponding initial (i.e. at the source position) spanwise-wavenumber spectra of the cross-flow instability waves are presented in *figure 4*. This figure shows that the theory is able to predict correctly the initial spectrum of the CF-waves excited by the surface vibrations in the swept-wing boundary layer, especially in a range of the spanwise wavenumber corresponded to the most unstable cross-flow modes which are close to $\beta'\delta_{1s} \approx 0.4$. (Prime at β denotes the (x', z')-coordinate system.) The observed agreement seems to be rather good taking into account the extreme complexity of the experimental procedure of determining the receptivity coefficients.

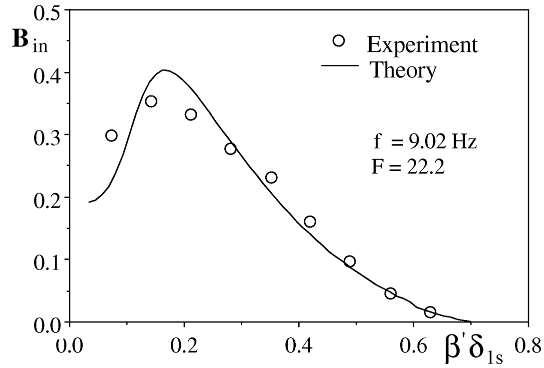

Figure 4. Experimental and theoretical initial spectra (amplitude parts) of CF-waves excited by vibrations in a swept-wing boundary layer (Crouch et al. [81,82]).

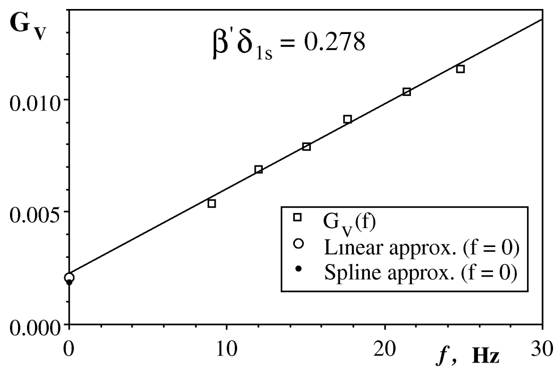

Figure 5. Experimental values of vibration receptivity coefficients for a swept-wing boundary layer versus frequency of vibrations for $\beta'\delta_{1s} = 0.278$ (Kachanov et al. [80]).

Some of results obtained in Kachanov et al. [80] for other frequencies of vibrations are illustrated in *figure 5* where an experimentally obtained dependence of the receptivity amplitude on the frequency is shown for one of the fixed values of the spanwise wavenumber. It is found that the receptivity coefficients increase monotonously with the frequency of vibration (both in experiment and theory). At low spanwise wavenumbers $\beta'\delta_{1s}$ (from 0.2 to about 0) and high frequencies the theory gives somewhat greater values of the initial amplitudes as compared to the experiment. However, and this is of great importance, in the range $\beta'\delta_{1s} \approx 0.3$ to 0.6, which includes the most unstable cross-flow modes (see Gaponenko et al. [59,60]), an agreement is very good in the whole frequency range studied, especially for the initial spectra of the CF-modes.

4. Receptivity of swept-wing boundary layer to surface roughness

The difficulties of the experimental study of the linear roughness receptivity problem, described above in section 2.3, were overcome in the experiments (Gaponenko et al. [79]). This was done in the following way. In these experiments a new method of obtaining the roughness-receptivity coefficients was developed and used. The method incorporates an investigation of the swept-wing boundary-layer receptivity to localized surface vibrations at several disturbance frequencies (as close to zero frequency as possible) and an extrapolation of the results obtained to the zero frequency of vibration. For most values of the spanwise wavenumber (except for very small ones) the extrapolation does not represent a problem because of a very simple behavior of the

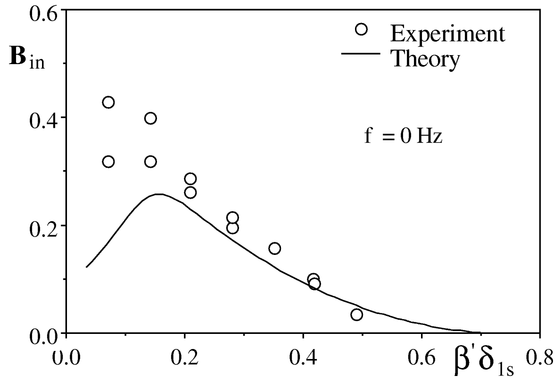

Figure 6. Experimental and theoretical initial spectra (amplitude parts) of CF-vortices excited by surface roughness in swept-wing boundary layer (Crouch et al. [81,82]).

receptivity functions with frequency (see, e.g., *figure 5*). The correctness of the limit transition $f \to 0$ was investigated and confirmed. As a result, the coefficients of the linear receptivity of the swept-wing boundary layer to the surface roughness were obtained experimentally for various values of the disturbance spanwise wavenumber.

A quantitative comparison of the experimental roughness receptivity characteristics with the theoretical ones obtained by Crouch (see Crouch [23,65]) was performed in Crouch et al. [81,82]. The experimental and theoretical initial spectra of the CF-vortices excited by a surface roughness are shown in *figure 6*. Despite some difficulties observed at low values of the spanwise wavenumber (both in theory and experiment), the theory is able to predict correctly the initial amplitudes of the CF-vortices in the range $\beta'\delta_{1s} \approx 0.3$ to 0.6, which includes the most unstable cross-flow vortices. Both in theory and experiment the values of the roughness receptivity coefficients turned out to be significantly lower than those of the vibration receptivity coefficients.

5. Three-dimensional acoustic receptivity

As was mentioned in section 2.5, the role of the acoustic waves in the swept-wing boundary-layer receptivity was not clear until recently, in particular due to some experimental difficulties. The first experimental quantitative information about the excitation of the CF-waves by acoustics was obtained very recently. It is discussed below in section 5.1.

5.1. Three-dimensional boundary layer on swept wing

The main problem of investigation of the 3D acoustic receptivity consists in a difficulty in measuring weak CF-instability waves on a background of rather intensive signals, conditioned by the acoustics itself, and the associated model and probe vibrations which have the same (acoustic) frequency. This problem has been solved in two experimental works, discussed in sections 5.1.1 and 5.1.2, in two different ways. The first of these experimental methods (section 5.1.1) gives also possibility to start an investigation of another important problem — a scattering of the acoustic wave on unsteady surface non-uniformities (i.e. on surface vibrations) in the 3D swept-wing boundary layer.

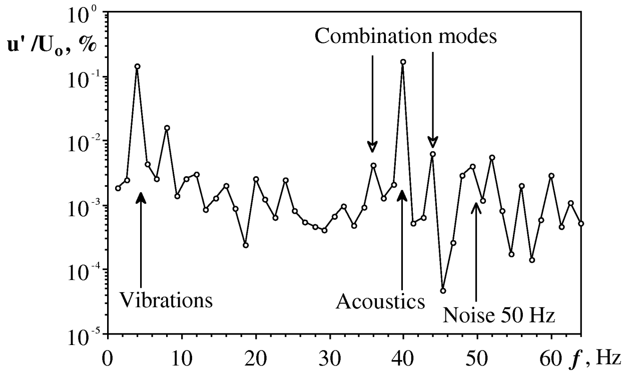

Figure 7. Typical frequency spectrum of linearized hot-wire signal measured in swept-wing boundary layer at scattering of an acoustic wave on localised surface vibrator (Ivanov et al. [84]).

5.1.1. Vibration-acoustic receptivity

The idea of the experimental method developed and used in Ivanov, Kachanov and Koptsev [84] was to 'tune-out' from the acoustic frequency, by means of making the surface roughness a non-stationary one, oscillating with a very low frequency. In this case the scattering of the acoustics attributed to the linear receptivity mechanism, as well as the excitation of the CF-instability waves, take place at two combination frequencies $f_{1,2} = f_a \pm f_v$. All characteristics of these weak waves can be relatively easy measured in this case by means of the frequency Fourier analysis. On the other hand, if the surface vibrator is nearly 'frozen' during one acoustic period (i.e. $f_v \ll f_a$) and the characteristic time scale of the vibrations $T_v = 1/f_v$ is much greater than time $T_d = d/U$ that is necessary for passing the flow above the non-uniformity with diameter d, then the receptivity mechanism is a quasi-stationary one and corresponds to a scattering of the acoustic wave on a surface roughness.

The experiments were conducted on the same swept-wing model as that described above in sections 3.3 and 4 but at a slightly higher free-stream speed. At the position of the vibrator the local Reynolds number was $\mathrm{Re} = 655$. The acoustic wave propagating upstream was excited at a frequency $f_a = 40$ Hz, while the vibrations were at $f_v = 4.0$ Hz. These frequencies corresponded to the frequency parameters $F = 42.1$ and 4.21 respectively. The circular surface vibrator used in this experiment was the same as in some previous studies (see section 3).

A typical frequency spectrum of the linearized hot-wire signal measured in the boundary layer downstream of the vibrator, is presented in *figure 7*. The largest peaks are observed at the frequencies of vibrations and acoustics. The combination modes $f_{1,2} = f_a \pm f_v$ are also clearly seen at the frequencies 36 and 44 Hz. Detailed measurements have shown that these modes corresponded to two CF-instability waves excited due to a linear receptivity mechanism attributed to the scattering of the acoustic wave on the surface vibrator. In particular, it was shown that the dispersion characteristics of these waves correspond to the dispersion functions $\alpha_r^* = \alpha_r^*(\beta^*)$ obtained on the same swept-wing model in previous experiments (Gaponenko et al. [60]) for the CF-waves. This fact is illustrated in *figure 8*. (The superscript '*' designates a local coordinate system coupled with the potential flow velocity at the boundary layer edge.)

Quantitative values of the acoustic receptivity coefficients were estimated in Ivanov et al. [84] integrally for the whole spanwise-wavenumber spectrum for the two combination modes. (It was impossible to obtain in these experiments the receptivity coefficients for every normal mode of the spanwise wavenumber spectrum

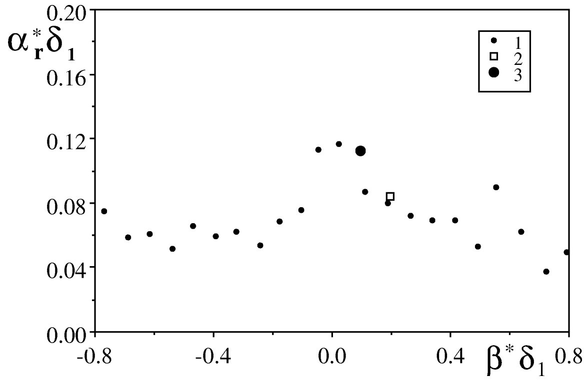

Figure 8. Comparison of instability wave dispersion characteristics measured in swept-wing boundary layer at $f = 35$ Hz. 1 — CF-instability modes (Gaponenko et al. [60]); 2 — most intensive CF-waves generated by vibrator only (Ivanov and Kachanov [69,70]); 3 — combination mode at scattering of acoustics on surface vibrator (Ivanov et al. [84]).

due to insufficient accuracy of the measurements.) This coefficient was defined as

$$G_{\mathrm{va}} \stackrel{\mathrm{df}}{=} \frac{A_{\mathrm{CF}}}{A_{\mathrm{v}} A_{\mathrm{a}}}, \tag{6}$$

where A_{a} is the non-dimensional amplitude of the velocity fluctuations in the acoustic wave, A_{v} is the non-dimensional amplitude of the vibrations, and A_{CF} is the non-dimensional amplitude of the velocity fluctuations in the combination wave (all normalized by the edge velocity and the boundary-layer displacement thickness). These values were: $A_{\mathrm{a}} = 0.15\%$, $A_{\mathrm{v}} = 5.7\%$ (58 μm), and $A_{\mathrm{CF}} \approx 0.0025\%$. The receptivity coefficients for the two combination modes were estimated as $G_{\mathrm{va}} \approx 0.29$.

An effectiveness of this acoustic receptivity mechanism was compared in Ivanov et al. [84] with that of the vibration receptivity mechanism in the following way. It was found that at the acoustic intensity of 110 dB the amplitude of the CF-wave produced due to the scattering of the acoustics on the vibrator is, by a factor of 180, less than the amplitude of the CF-wave generated directly by the same vibrator (without the acoustic wave) oscillating at the acoustic frequency with the same amplitude. These two CF-amplitudes become equal to each other at the acoustic intensity about 155 dB. This observation is consistent with the theoretical result (Crouch [65]) and shows that the acoustic receptivity mechanism is relatively weaker than the vibration (or roughness) receptivity mechanism. However it is necessary to note that: (i) the mechanism of the acoustic receptivity does exist and (ii) the mechanism of the vibration-acoustic receptivity can play a significant role in the swept-wing boundary-layer transition because it leads to a redistribution of the disturbance energy in the frequency-wavenumber spectrum. Indeed, if the acoustic wave spectrum and the vibration spectrum are out of range of the unstable CF-modes the vibration receptivity mechanism is not at all significant, while the vibration-acoustic receptivity mechanism can be important if the combination spectral modes correspond to unstable CF-waves.

5.1.2. Roughness-acoustic receptivity

Another idea of a method of the qualitative experimental investigation of the 3D acoustic receptivity was developed and used in (Ivanov, Kachanov and Koptsev [85]) for the case of a swept-wing boundary layer. This idea consists in a simplification of the problem by means of designing a special roughness that would generate (in the presence of the acoustics) a single normal CF-instability mode inclined at a certain (desirable) angle to the flow direction, i.e. with a definite value of the spanwise wavenumber β_1. Based on previous 2D-flow

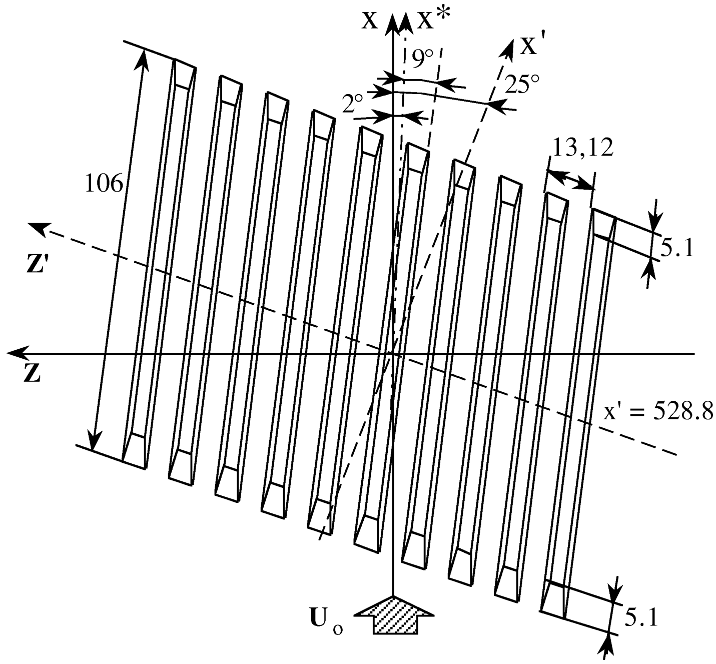

Figure 9. Sketch of the phased roughness used in (Ivanov et al. [84]) for experimental study of roughness-acoustic receptivity of a swept-wing boundary layer.

experience (see, e.g., Aizin and Polyakov [7] and Kosorygin et al. [41]) it was expected that such roughness could be easily arranged in a completely uniform flow in a form of a strip pasted onto the surface at a necessary angle. However (i) the swept-wing boundary layer is not uniform, (ii) the typical inclination angle of such a strip has to be very large for the most unstable CF-mode (around 85°) that suggests a very big length of the strip if we want to obtain a real normal mode that would be uniform along the span rather than localized along the span, and (iii) a streamwise length of the strip has to be short enough if we want to investigate the localized (along the chord) receptivity. These three circumstances make the problem quite difficult. A 'phased roughness' designed and used in experiments (Ivanov, Kachanov and Koptsev [85]) gives the possibility of solving this problem and of obtaining quantitative experimental data on the localized roughness-acoustic receptivity of the swept-wing boundary layer.

A sketch of the phased roughness, produced with the help of a specially developed technology, is shown in *figure 9* (plan view). The spanwise period of every roughness element (i.e. every strip) $\lambda'_z = 13.1$ mm corresponded to the spanwise wavenumber $\beta'_1 = 2\pi/\lambda'_z = 0.480$ rad/mm ($\beta'_1\delta_{1r} = 0.506$) of the CF-instability wave whose excitation was to be investigated (where δ_{1r} is the boundary layer thickness in the center of the roughness). This wave was rather close to the most unstable CF-mode. The axis of each roughness strip was inclined to the x-axis (i.e. to the axis of the wind-tunnel) at angle $\xi = 90 - \theta^*_1 - \gamma = 9°$. For the CF-wave with the spanwise wavenumber $\beta' = \beta'_1$ this angle corresponded to propagation angle $\theta^*_1 = 83.0°$ (with respect to the local potential-flow velocity vector). Here $\gamma = -2°$ is a local yaw angle of the potential flow with respect to the x-axis. The length of the roughness elements $\lambda^*_x = 107.2$ mm corresponded to the streamwise wavenumber of the same CF-instability mode $\alpha^*_{r1} = 0.059$ rad/mm ($\alpha^*_{r1}\delta_{1r} = 0.0617$). The values of θ^*_1 and α^*_{r1} were taken from the results of the CF-stability measurements (Gaponenko et al. [59,60]). The height of the roughness was about 90 μm. The harmonic acoustic wave had frequency 42 Hz and propagated upstream from a loudspeaker.

The measurements performed in (Ivanov et al. [85]) and the results of the data processing have shown that the acoustic wave does excite on the phased roughness a single (in practice) normal CF-instability wave inclined at the rated (desirable) angle to the flow direction. The initial amplitude and phase of this mode were determined at the position of the roughness center. The shape of the phased roughness was accurately measured and subjected to double Fourier transform (the Fourier series in the spanwise direction and the Fourier integral in the streamwise direction). The amplitude and phase of the resonant mode in this spectrum were determined with the help of the dispersion characteristics of the CF-wave excited in the boundary layer. Finally, the values of the amplitude and phase of the complex roughness-acoustic receptivity coefficient have been obtained experimentally.

Following the theoretical studies (see, e.g., Crouch [65]) the complex 'roughness-acoustic receptivity coefficient'

$$G_{\mathrm{rac}}(\beta') = G_{\mathrm{ra}}(\beta')\exp\left[i\varphi_{\mathrm{ra}}(\beta')\right] \tag{7}$$

was defined in Ivanov et al. [85] (for a fixed frequency of the acoustics) as

$$G_{\mathrm{rac}}(\beta') \stackrel{\mathrm{df}}{=} \frac{A_{\mathrm{inc}}(\beta')}{\tilde{H}_{\mathrm{rc}}(\beta')A_{\mathrm{ac}}}, \tag{8}$$

where

$$A_{\mathrm{inc}}(\beta') = A_{\mathrm{in}}(\beta')\exp\left[i\varphi_{\mathrm{in}}(\beta')\right] \tag{9}$$

is the complex initial spectral (Fourier-series) amplitude of the CF-wave excited by acoustics on the roughness,

$$\tilde{H}_{\mathrm{rc}}(\beta') = \tilde{H}_{\mathrm{r}}(\beta')\exp\left[i\tilde{\gamma}_{\mathrm{r}}(\beta')\right] \tag{10}$$

is the complex spectral (double Fourier-series-integral) amplitude of the roughness shape determined for the resonant mode of this spectrum, and

$$A_{\mathrm{ac}} = A_{\mathrm{a}}\exp(i\varphi_{\mathrm{a}}) \tag{11}$$

is the complex amplitude of the velocity fluctuations in the acoustic wave measured above the roughness in the free stream. Note that the acoustic wavenumber does not play any role because it equals to zero, in practice. This corresponds to the fact that the acoustic field is practically uniform in the region of the CF-wave excitation, which has a characteristic scale of the surface vibrator. Definition (8) of the roughness-acoustic receptivity coefficient means that for the receptivity amplitude and phase we have respectively

$$G_{\mathrm{ra}}(\beta') = \frac{A_{\mathrm{in}}(\beta')}{\tilde{H}_{\mathrm{r}}(\beta')A_{\mathrm{a}}} \quad \text{and} \quad \varphi_{\mathrm{ra}}(\beta') = \varphi_{\mathrm{in}}(\beta') - \tilde{\gamma}_{\mathrm{r}}(\beta') - \varphi_{\mathrm{a}}. \tag{12}$$

All dimensional values were normalized in (Ivanov et al. [85]) by the potential flow velocity and the boundary-layer displacement thickness at the position of the roughness.

For the rated frequency-wavenumber mode it was found in (Ivanov et al. [85]) that $A_{\mathrm{in}} = 0.024\%$, $\varphi_{\mathrm{in}} = -46.6^\circ$; $\tilde{H}_{\mathrm{r}} = 5.65$, $\tilde{\gamma}_{\mathrm{r}} = 0$; and $A_{\mathrm{a}} = 0.15\%$, $\varphi_{\mathrm{r}} = -15.6^\circ$. The amplitude and phase of the roughness-acoustic receptivity coefficient for the swept-wing boundary layer were found to be $G_{\mathrm{ra}} \approx 0.030$ and $\varphi_{\mathrm{ra}} = -31^\circ$ respectively. The value of the roughness-acoustic receptivity amplitude G_{ra} can not be compared directly with that of the vibration-acoustic receptivity amplitudes $G_{v\mathrm{a}}$ found in section 5.1.1 due to their different definitions. However, these two coefficients can be used for verification of the linear acoustic receptivity theory for the swept-wing boundary layer.

5.2. Two-dimensional boundary layer on an airfoil

The first quantitative experimental study of the 3D acoustic receptivity of two-dimensional boundary layer was performed very recently in Würz et al. [86] on an airfoil. A comparison of these results with the direct numerical simulation (DNS) was carried out in Würz et al. [87].

The experiments were carried out in the Laminar Wind Tunnel of the Institute of Aerodynamics and Gasdynamics of Stuttgart University at a turbulence level less than 0.02%. The measurements were performed on a symmetrical airfoil section at zero angle of attack and at a Reynolds number of $1.2 \cdot 10^6$ based on the arc-length 0.615 m measured from the leading edge. Following the idea of the experiments (Ivanov et al. [84]) an unsteady surface non-uniformity oscillating with a very low frequency ($f_v = 17.0$ Hz) was also used in experiments (Würz et al. [86,87]) providing a quasi-steady scattering of the acoustic wave exited by a loudspeaker at frequency $f_a = 64 f_v = 1088$ Hz (with an amplitude about 100 dB). The non-uniformity represented a circular surface vibrator (6 mm in diameter) positioned near the first branch of the neutral stability curve for the 2D TS-waves and oscillated with an amplitude about 33 μm r.m.s. Similarly to the experiments described in section 5.1.1, the linear receptivity mechanism provided an excitation of the 3D instability waves (the TS-waves in the present case) at two combination frequencies $f_{1,2} = f_a \pm f_v$. The spatial behavior of these perturbations was investigated in detail by means of a hot-wire anemometer. The wave-trains were decomposed to normal oblique TS-waves and their initial (at the position of the vibrator) amplitudes and phases were determined with the help of an extrapolation procedure using the linear stability theory results. To obtain the acoustic receptivity coefficients the shape of the surface vibrator was also studied accurately and decomposed to normal oblique vibration waves.

The DNS was performed in Würz et al. [87] for a steady roughness, i.e. for the frequency of vibration exactly equaled to zero. Taking into account a quasi-steady character of the acoustic scattering on the vibrator in the described experiments (because the conditions $f_v \ll f_a$ and $T_d \ll T_v$ mentioned in section 5.1.1 were satisfied very well) the receptivity mechanism studied by means of the DNS correspond almost exactly to the mechanism studied experimentally. Both in the DNS and experiments the same definition of the acoustic receptivity function was used.

Thus, for a fixed frequency of the acoustic wave and infinitely low frequency of the surface vibrations the 'complex roughness-acoustic receptivity function'

$$G_{rac}(\beta) = G_{ra}(\beta) \exp\left[i\varphi_{ra}(\beta)\right] \tag{13}$$

was defined in Würz et al. [87] as

$$G_{rac}(\beta) \stackrel{df}{=} \frac{B_{inc}(\beta)}{\tilde{C}_{rc}(\beta) A_{ac}}, \tag{14}$$

where

$$B_{inc}(\beta) = B_{in}(\beta) \exp\left[i\phi_{in}(\beta)\right] \tag{15}$$

is the complex initial spectrum (Fourier-integral) of the TS-wave excited by acoustics on the surface non-uniformity,

$$\tilde{C}_{rc}(\beta) = \tilde{C}_{r}(\beta) \exp\left[i\tilde{\lambda}_{r}(\beta)\right] \tag{16}$$

is the complex spectrum (double Fourier-integral) of the non-uniformity shape determined for the resonant modes corresponded to the dispersion function $\alpha_r = \alpha_r(\beta)$ for the 3D TS-waves, and

$$A_{ac} = A_a \exp(i\varphi_a) \tag{17}$$

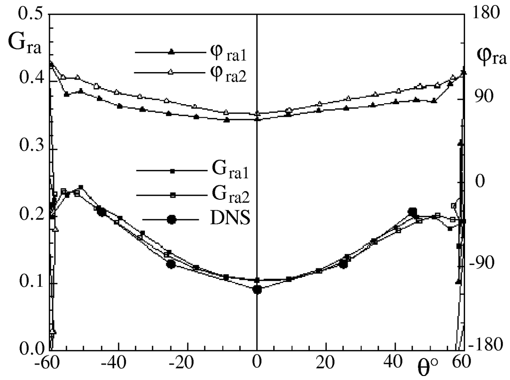

Figure 10. Amplitude and phase parts of the complex receptivity coefficients for excitation of 3D TS-waves by acoustics on localised surface non-uniformities in an airfoil boundary layer versus TS-wave propagation angle (Würz et al. [87]).

is the complex amplitude of the velocity fluctuations in the acoustic wave determined in the free stream above the non-uniformity. Similar to the case of the acoustic receptivity of the swept-wing boundary layer (see section 5.1.2) the acoustic wavenumber did not play in Würz et al. [87] any role in the definition of the receptivity function (14). This is because the acoustic wave (with a wavelength about 300 mm) produced practically uniform instantaneous perturbation within the region of the TS-wave excitation that had a characteristic scale of the surface vibrator (6 mm). The definition (14) of the acoustic receptivity function leads to the following expressions for the amplitude and phase parts of this function respectively

$$G_{\mathrm{ra}}(\beta) = \frac{B_{\mathrm{in}}(\beta)}{\tilde{C}_{\mathrm{r}}(\beta) A_{\mathrm{a}}} \quad \text{and} \quad \varphi_{\mathrm{ra}}(\beta) = \phi_{\mathrm{in}}(\beta) - \tilde{\lambda}_{\mathrm{r}}(\beta) - \varphi_{\mathrm{a}}. \tag{18}$$

Similar to the previous cases discussed above all dimensional values were normalized in Würz et al. [86,87] by the potential flow velocity and the boundary-layer displacement thickness at the position of the surface non-uniformity.

The main result of the paper Würz et al. [87] is presented in *figure 10* where the amplitude and phase parts of the experimental acoustic receptivity coefficients are shown versus the wave propagation angle θ for the two combination modes excited by the acoustics on the low-frequency vibrator. The corresponding amplitude receptivity function obtained by means of the DNS for the zero frequency of vibration is also shown in *figure 10*. The acoustic receptivity of the studied 2D boundary layer is seen to be greater for the 3D surface non-uniformities rather than for the 2D ones. The receptivity phases are weakly dependent on the wave propagation angle. A very good agreement between the experimental and DNS results is observed.

6. Concluding remarks

At present the problem of linear three-dimensional receptivity of the subsonic incompressible boundary layers has been investigated quite extensively; however some significant receptivity mechanisms still have not been studied. The most complete experimental results are obtained for the surface vibration receptivity of the two-dimensional boundary layers (including an adverse pressure gradient case) and the 3D (swept-wing) boundary layer, as well as for the surface roughness receptivity of the swept-wing boundary layer. In these cases the receptivity coefficients, independent of the specific shape of the vibrations, are obtained experimentally and compared with the linear receptivity theories. Similarly, very detail results are available for the roughness acoustic receptivity of a 2D boundary layer on an airfoil for the case of positioning the roughness elements in a vicinity of the neutral stability point for 2D Tollmien–Schlichting waves, where the basic flow is rather close to the Blasius boundary layer. The latter results are compared with those obtained by means of the direct numerical simulation.

The acoustic receptivity of the 3D boundary layers on swept wings has not yet been studied in such detail. However, the experimental results show that the acoustic perturbations do excite the traveling cross-flow instability waves in presence of a surface roughness or vibration. For the roughness acoustic receptivity, the receptivity coefficients independent of the roughness shape are obtained experimentally.

Summarizing the results of the studies discussed above the following main conclusions can be drawn.

1. The 2D incompressible boundary layers are more receptive to 3D perturbations rather than to 2D ones for all the kinds of receptivity mechanisms studied at present. Together with greater growth rates of the 3D TS-waves, observed in the vicinity of the lower branch of the neutral stability curve in previous experimental and theoretical studies (which take into account the flow non-parallelism), this fact suggests that the 3D instability waves can dominate in the subsonic boundary layers (in contrast to a common belief) even in cases when the 2D surface non-uniformities have the same order of magnitude as that of the 3D non-uniformities.

2. The adverse pressure gradient leads to a very significant decrease of the surface receptivity coefficients, especially for the 3D modes. This compensates partially the flow destabilization observed within the framework of the linear stability theory. The reduction of the receptivity coefficients in the 2D APG boundary layers is observed both in theory and experiment for surface vibrations and blowing-suction disturbances originating from the wall. Theory also predicts a significant increase of the surface receptivity coefficients in presence of a favorable pressure gradient. Thus, the receptivity and stability mechanisms are in a strong competition with each other in the 2D boundary layers with streamwise pressure gradients.

3. The swept-wing boundary layers are significantly more receptive to the unsteady surface non-uniformities (i.e. to vibrations) rather to steady non-uniformities (i.e. to the roughness or waviness). This fact is observed both in theory and experiment. For the most unstable cross-flow waves the surface receptivity coefficient can be several times greater than those for the cross-flow vortices. In view of a relatively weak acoustic receptivity coefficients the surface vibrations can play an important role in the excitation of the traveling cross-flow instability waves in the swept-wing boundary layer. Note however that the free-stream vortex receptivity coefficients of the swept-wing boundary layer are not yet investigated properly, especially by experiments.

4. The vibration-acoustic receptivity mechanism is studied at present only in the case of the quasi-stationary scattering of the acoustic wave (both in the 2D and 3D boundary layers). At the same time, the non-stationary vibration-acoustic receptivity mechanism can be significant, especially in 2D boundary layers because, in particular, it can provide a redistribution of the external disturbance energy in the frequency spectrum. Even when the main energy of the acoustic and vibration perturbations is concentrated in the frequency ranges corresponding to stable boundary-layer perturbations, the combination modes excited due to this receptivity mechanism can have characteristic frequencies corresponding to the unstable waves.

5. In all the cases discussed above the receptivity theories and the DNS results are able to predict correctly the quantitative values of the receptivity coefficients obtained in experiment. These theoretical approaches can be used in the modern advanced methods of transition prediction based on accurate physical notions about the processes of turbulence origin in the boundary layers.

Acknowledgements

This work was supported by Russian Foundation for Basic Research (grants No 97-01-00638 and 98-01-04090) German Research Council (DFG), and Boeing International Inc.

References

[1] Morkovin M.V., Critical evaluation of transition flow laminar to turbulent shear layers with emphasis of hypersonically traveling bodies, AFFDL TR, 68–149, 1968.
[2] Kachanov Y.S., Kozlov V.V., Levchenko V.Y., Generation and development of small disturbances in laminar boundary layer under the action of acoustic fields, Izv. Sib. Otd. Akad. Nauk USSR, Ser. Tekh. Nauk. 13 (3) (1975) 18–26 (in Russian).
[3] Kachanov Y.S., Kozlov V.V., Levchenko V.Y., Origin of Tollmien–Schlichting waves in boundary layer under the influence of external disturbances, Izv. Akad. Nauk USSR, Mekh. Zhidk. i Gaza 5 (1978) 85–94 (in Russian). (See translation into English in Fluid Dyn. 13 (1979) 704–711.)
[4] Kachanov Y.S., Kozlov V.V., Levchenko V.Y., Onset of Turbulence in Boundary Layers, Nauka Pub., Novosibirsk, 1982 (in Russian).
[5] Shapiro P.J., The influence of sound upon laminar boundary layer instability, MIT Acoustic and Vibration Lab. Rep. 83483-83560-1, 1977.
[6] Kachanov Y.S., Kozlov V.V., Levchenko V.Y., Maksimov V.P., Transformation of external disturbances into the boundary layer waves, in: Proc. Sixth Int. Conf. on Numerical Methods in Fluid Dyn., Springer, Berlin, 1979, pp. 299–307.
[7] Aizin L.B., Polyakov N.F., Acoustic generation of Tollmien–Schlichting waves over local unevenness of surface immersed in stream, Preprint No 17, USSR Acad. Sci., Sib. Div., Inst. Theor. Appl. Mech., Novosibirsk, 1979 (in Russian).
[8] Leehey P., Influence of environment in laminar boundary layer control, in: Hough G. (Ed.), Viscous Flow Drag Reduction, Vol. 72, 1980, pp. 4–16.
[9] Mangur C.J., On the sensitivity of shear layers to sound, AIAA Paper No 77-1369, 1977.
[10] Tam C.K.W., Excitation of instability waves in a two-dimensional shear layer by sound, J. Fluid Mech. 89 (2) (1978) 357–371.
[11] Murdock J.W., The generation of Tollmien–Schlichting wave by a sound wave, P. Roy. Soc. Lond. A Mat. 372 (1980) 1517.
[12] Nishioka M., Morkovin M.V., Boundary-layer receptivity to unsteady pressure gradients: experiments and overview, J. Fluid Mech. 171 (1986) 219–261.
[13] Loehrke R. I., Morkovin M.V., Fejer A.A., Review. — Transition in nonreversing oscillating boundary layer, J. Fluid Eng.-T. ASME 97 (4) (1975) 534–549.
[14] Reshotko E., Boundary-layer stability and transition, Annu. Rev. Fluid Mech. 8 (1976) 311–349.
[15] Morkovin M.V., Instability, transition to turbulence and predictability, AGARD-AG-236, 1977.
[16] Zhigulyov V.N., Tumin A.M., Origin of Turbulence, Nauka Pub., Novosibirsk, 1987 (in Russian).
[17] Goldstein M.E., Hultgren L.S., Boundary-layer receptivity to long-wave free-stream disturbances, Ann. Rev. Fluid Mech. 21 (1989) 137–166.
[18] Kerschen E.J., Boundary layer receptivity, AIAA Paper No. 89-1109, 1989.
[19] Kerschen E.J., Choudhari M., Heinrich R.A., Generation of boundary layer instability waves by acoustic and vortical freestream disturbances, in: Arnal D., Michel R. (Eds), Laminar-Turbulent Transition, Springer, Berlin, 1990, pp. 477–488.
[20] Kozlov V.V., Ryzhov O.S., Receptivity of boundary layers: asymptotic theory and experiment, P. Roy. Soc. Lond. A Mat. 429 (1990) 341–373.
[21] Morkovin M.V., Reshotko E., Dialogue on progress and issues in stability and transition research, in: Arnal D., Michel R. (Eds), Laminar-Turbulent Transition, Springer, Berlin, 1990, pp. 3–29.
[22] Choudhari M, Streett C.L., A finite Reynolds number approach for the prediction of boundary-layer receptivity in localized regions, Phys. Fluids A 4 (1992) 2495–2514.
[23] Crouch J.D., Theoretical studies on the receptivity of boundary layers, AIAA Paper 94-2224, 1994.
[24] Choudhari M., Roughness-induced generation of crossflow vortices in three-dimensional boundary layers, Theor. Comput. Fluid Dynamics 6 (1) (1994) 1–30.
[25] Dietz A.J., Local boundary-layer receptivity to a convected free-stream disturbances, J. Fluid Mech. 378 (1999) 291–317.
[26] Schubauer G.B., Skramstad H.K., Laminar boundary-layer oscillations and transition on a flat plate, J. Res. Natl. Bur. Stan. 38 (1947) 251–292.
[27] Gaster M., On the generation of spatially growing waves in a boundary layer, J. Fluid Mech. 22 (1965) 433–441.
[28] Gilyov V.M., Kozlov V.V., Excitation of Tollmien–Schlichting waves in a boundary layer on a vibrating surface, Zhur. Prik. Mekh. Tekh. Fiz. 6 (1984) 73–77 (in Russian).
[29] Terent'ev E.D., A linear problem on a vibrator in subsonic boundary layer, Prik. Matem. Mekh. 45 (6) (1981) 1049–1055 (in Russian).
[30] Tumin A.M., Fyodorov A.V., Generation of instability waves in a boundary layer on a vibrating surface, Zhur. Prik. Mekh. Tekh. Fiz. 3 (1983) 72–79 (in Russian).
[31] Tumin A.M., Fyodorov A.V., Generation of instability waves in boundary layer by localized vibrator, Zhur. Prik. Mekh. Tekh. Fiz. 6 (1984) 65–72 (in Russian).
[32] Terent'ev E.D., A linear problem on a vibrator oscillating harmonically at supercritical frequencies in a subsonic boundary layer, Prik. Matem. Mekh. 48 (2) (1984) 264–272 (in Russian).

[33] Fyodorov A.V., Generation of Tollmien–Schlichting waves in a boundary layer by means of a periodic external perturbation localized on the surface, Izv. AN SSSR. Mekh. Zhidk. i Gaza. 6 (1984) 36–41 (in Russian).
[34] Michalke A., Receptivity of axisymmetric boundary layers due to excitation by a Dirac point source at the wall, Eur. J. Mech. B/Fluids 14 (4) (1995) 373–393.
[35] Michalke A., Excitation of a 3D-wavetrain by a Dirac point source at the wall and its growth in a decelerated laminar boundary layer, Eur. J. Mech. B/Fluids 16 (6) (1997) 779–795.
[36] Michalke A., Neemann K., Excitation of instability waves in wall boundary layers with adverse pressure gradients by various types of Dirac sources, Acta Mech. 122 (1997) 33–48.
[37] Squire H.D., On the stability for three-dimensional disturbances of viscous fluid between parallel walls, P. Roy. Soc. Lond. A Mat. 142 (1933) 621–628.
[38] Kachanov Y.S., Obolentseva T.G., A method of study of influence of the flow nonparallelism on the 3D stability of Blasius boundary layer, in: 8th Int. Conference on Methods of Aerophysical Research. Proceedings. Part II, Inst. Theor. and Appl. Mech., Novosibirsk, 1996, pp. 100–105.
[39] Kachanov Y.S., Obolentseva T.G., Development of three-dimensional disturbances in the Blasius boundary layer. 3. Nonparallelism effects, Thermophysics and Aeromechanics 5 (3) (1998) 331–338.
[40] Bertolotti F.P., Linear and nonlinear stability of boundary layers with streamwise varying properties, PhD thesis, Ohio State University, USA, 1991.
[41] Kosorygin V.S., Levchenko V.Y., Polyakov N.F., On generation and evolution of waves in laminar boundary layer, in: Kozlov V.V. (Ed.), Laminar-Turbulent Transition, Springer, Berlin, 1985, pp. 233–242.
[42] Kosorygin V.S., Experimental investigation of laminar boundary layer at low natural and acoustic perturbations, PhD thesis, Inst. Theor. and Appl. Mech., Novosibirsk, 1986 (in Russian).
[43] Saric W.S., Hoos J.A., Kohama Y., Boundary-layer receptivity: part 1: Freestream sound and 2D roughness strips, Report, CEAS-CR-R-90191, Arizona State University, College of Engineering and Applied Sciences, Tempe, 1990.
[44] Wiegel M, Wlezien R.W., Acoustic receptivity of laminar boundary layers over wavy walls, AIAA Paper 93-3280, 1993.
[45] Zhou M.D., Liu D.P., Blackwelder R.F., An experimental study of receptivity of acoustic waves in laminar boundary layers, Exp. Fluids 17 (1994) 1–9.
[46] Kosorygin V.V., Radeztsky R.H., Saric, W.S., Laminar boundary layer sound receptivity and control, in: Kobayashi R. (Ed.), Laminar-Turbulent Transition, Springer, Berlin, 1995, pp. 417–524.
[47] Choudhari M., Kerschen E.J., Instability wave patterns generated by interaction of sound wave with three-dimensional wall suction or roughness, AIAA Paper 90-0119, 1990.
[48] Tadjfar M., Bodonyi R.J., Receptivity of laminar boundary layer to the interaction of a three dimensional roughness element with time-harmonic free-stream disturbances, J. Fluid Mech. 242 (1992) 701–720.
[49] Tadjfar M., Receptivity of laminar boundary layer to the interaction of a three dimensional roughness element with time-harmonic free-stream disturbances, PhD Diss, Dept. Aero. and Astro. Eng., Ohio State University, 1990.
[50] Zhigulyov V.N., On excitation and development of instability in three-dimensional steady boundary layers, Zhur. Prik. Mekh. Tekh. Fiz. 4 (1983) 100–110 (in Russian).
[51] Tumin A.M., Zhigulyov V.N., Excitation of Tollmien–Schlichting waves in a boundary layer on vibrating surface of an infinite swept wing, Zhur. Prik. Mekh. Tekh. Fiz. 5 (1983) (in Russian).
[52] Michel R., Arnal D., Coustols E., Juillen J.C., Experimental and theoretical studies of boundary layer transition on a swept infinite wing, in: Kozlov V.V. (Ed.), Laminar-Turbulent Transition, Springer, Berlin, 1985, pp. 553–561.
[53] Dallmann U., Bieler H., Analysis and simplified prediction of primary instability of three-dimensional boundary layer flows, AIAA Paper No. 87-1337, 1987.
[54] Reed H.L., Saric W.S., Stability of three-dimensional boundary layers, Annu. Rev. Fluid Mech. 21 (1989) 235–284.
[55] Kachanov Y.S., Tararykin O.I., Fyodorov A.V., Investigation of stability to stationary boundary-layer disturbances in a model of a swept wing, Izv. Sib. Otd. Akad. Nauk SSSR, Ser. Tekh. Nauk. 5 1990 11–21 (in Russian).
[56] Bippes H., Instability features appearing on swept wing configurations, in: Arnal D., Michel R. (Eds), Laminar-Turbulent Transition, Springer, Berlin 1990, pp. 419–430.
[57] Kachanov Y.S., Tararykin O.I., The experimental investigation of stability and receptivity of a swept-wing flow, in: Arnal D., Michel R. (Eds), Laminar-Turbulent Transition, Springer, Berlin, 1990, pp. 499–509.
[58] Kohama Y., Ohta F., Segawa K., Development and interactions of instabilities in the crossflow field, in: Arnal D., Michel R. (Eds), Laminar-Turbulent Transition, Springer, Berlin, 1990, pp. 431–440.
[59] Gaponenko V.R., Ivanov A.V., Kachanov Y.S., Experimental study of cross-flow instability of a swept-wing boundary layer with respect to traveling waves, in: Kobayashi R. (Ed.), Laminar-Turbulent Transition, Springer, Berlin, 1995, pp. 373–380.
[60] Gaponenko V.R., Ivanov A.V., Kachanov Y.S., Experimental study of a swept-wing boundary-layer stability with respect to unsteady disturbances, Thermophysics Aeromechanics 2 (4) (1995) 287–312.
[61] Kachanov Y.S., Experimental studies of three-dimensional instability of boundary layers, AIAA Paper 96-1978, 1996.
[62] Radeztsky R.H., Reibert M.S., Saric W.S., Takagi S., Effect micron-sized roughness on transition in swept-wing flows, AIAA Paper 93-0076, 1993.
[63] Fyodorov A.V., Excitation of cross-flow instability waves in boundary layer on a swept-wing, Zhur. Prik. Mekh. Tekh. Fiz. 5 (1988) 46–52 (in Russian).

[64] Manuilovich S.V., Disturbances of three-dimensional boundary layer generated by surface roughness, Fluid Dyn. 24 (1990) 764–769.

[65] Crouch J.D., Receptivity of three-dimensional boundary layers, AIAA Paper 93-0074, 1993.

[66] Bertolotti F.P., On the birth and evolution of disturbances in three-dimensional boundary layers, in: Duck P.W., Hall P. (Eds), Nonlinear Instability and Transition in Three-Dimensional Boundary Layers, Kluwer, Dordrecht, 1996, pp. 247–256.

[67] Deyhle H., Bippes H., Disturbance growth in an unstable three-dimensional boundary layer and its dependence on environmental conditions, J. Fluid Mech. 316 (1996) 73–113.

[68] Ivanov A.V., Development of a wave packet in a boundary layer on a swept wing, Diploma Work (Masters Thesis), Novosibirsk State Tech. Univ., Novosibirsk, 1992 (in Russian).

[69] Ivanov A.V., Kachanov Y.S., A method of study of the stability of 3D boundary layers using a new disturbance generator, in: 7th Int. Conference on Methods of Aerophysical Research. Proceedings. Part 1, Inst. Theor. and Appl. Mech., Novosibirsk, 1994, pp. 125–130.

[70] Ivanov A.V., Kachanov Y.S., Excitation and development of spatial packets of instability waves in a three-dimensional boundary layer, Thermophysics Aeromechanics 1 (4) (1994) 287–305.

[71] Müler B., Bippes H., Experimental study of instability modes in a three-dimensional boundary layer, in: Proc. AGARD Symp. on Fluid Dynamics of Three-Dimensional Turbulent Shear Flows and Transition, AGARD-CP-438, 1988.

[72] Takagi S., Saric W.S., Radeztsky R.H., Spencer S.A., Orr D.J., Effect of sound and micro-sized roughness on crossflow dominated transition, B. Am. Phys. Soc. 36 (1991) 2630.

[73] Ivanov A.V., Kachanov Y. S., Obolentseva T.G., Experimental investigation of flat-plate boundary-layer receptivity to 3D surface vibrations, in: Stability and Transition of Boundary-Layer Flows, EUROMECH Colloquium 359. Collection of Abstracts, Universität Stuttgart, 10–13 March, 1997, abstract 4.

[74] Ivanov A.V., Kachanov Y.S., Obolentseva T.G., Michalke A., Receptivity of the Blasius boundary layer to surface vibrations. Comparison of theory and experiment, in: 9th Int. Conference on Methods of Aerophysical Research. Proceedings. Part I, Inst. Theor. and Appl. Mech., Novosibirsk, 1998, pp. 93–98.

[75] Kachanov Y.S., Michalke A., Three-dimensional instability of flat-plate boundary layers: Theory and experiment, Eur. J. Mech. B/Fluids 13 (4) (1994) 401–422.

[76] Michalke A., Al-Maaitan A.A., On the receptivity of the unstable wall boundary layer along a surface hump excited by 2-D Dirac source at the wall, Eur. J. Mech. B/Fluids 11 (1992) 521–542.

[77] Kachanov Y.S., Koptsev D.B., Smorodsky B.D., Receptivity of self-similar 2D boundary layer with adverse pressure gradient to 3D surface vibrations. Theory and experiment, in: International Workshop on Stability of Flows of Homogeneous and Heterogeneous Fluids. Abstracts, Inst. Theor. Appl. Mech., Novosibirsk, 20–23 April, 1999.

[78] Neemann K., Theoretische Untersuchungen zur Anregbarkeit instabiler Wellen in kompressiblen Wandgrenzschichten, PhD Diss., Technische Universität Berlin, Berlin, 1998.

[79] Gaponenko V.R., Ivanov A.V., Kachanov Y.S., Experimental study of 3D boundary-layer receptivity to surface vibrations, in: Duck P.W., Hall P. (Eds), Nonlinear Instability and Transition in Three-Dimensional Boundary Layers, Kluwer, Dordrecht, 1996, pp. 389–398.

[80] Kachanov Y.S., Gaponenko V.R., Ivanov A.V., Experimental study of swept-wing boundary-layer receptivity to stationary and non-stationary surface non-uniformities, in: Stability and Transition of Boundary-Layer Flows, EUROMECH Colloquium 359. Collection of Abstracts, Universität Stuttgart, 10–13 March, 1997, abstract 5.

[81] Crouch J.D., Gaponenko V.R., Ivanov A.V., Kachanov Y.S., Theoretical and experimental comparisons for the stability and receptivity of swept-wing boundary layers, B. Am. Phys. Soc. 42 (1997) 2174.

[82] Crouch J.D., Gaponenko V.R., Ivanov A.V., Kachanov Y.S., A method of experimental determination of the linear receptivity coefficients of a 3D boundary layer subjected to microscopic surface non-uniformities. Verification of theory, in: 9th Int. Conference on Methods of Aerophysical Research. Proceedings. Part II, Inst. Theor. and Appl. Mech., Novosibirsk, 1998, pp. 30–35.

[83] Kachanov Y.S., Tararykin O.I., Fyodorov A.V., Experimental simulation of swept-wing boundary layer in the region of secondary flow formation, Izv. Sib. Otd. Akad. Nauk SSSR, Ser. Tekh. Nauk. (1989) 44–53 (in Russian).

[84] Ivanov A.V., Kachanov Y.S., Koptsev D.B., An experimental investigation of instability wave excitation in three-dimensional boundary layer at acoustic wave scattering on a vibrator, Thermophysics Aeromechanics 4 (4) (1997) 359–372.

[85] Ivanov A.V., Kachanov Y.S., Koptsev D.B., Method of phased roughness for determining the acoustic receptivity coefficients, in: 9th Int. Conference on Methods of Aerophysical Research. Proceedings. Part II, Inst. Theor. and Appl. Mech., Novosibirsk, 1998, pp. 89–94.

[86] Würz W., Herr S., Wagner S., Kachanov Y.S., Experimental investigation on 3D acoustic receptivity of a laminar boundary layer in the presence of surface non-uniformities, in: Notes on Numerical Fluid Mechanics, Proc. 11 AG Stab. Symposium, Vieweg-Verlag, Berlin, 10–12 November, 1998 (accepted for publication).

[87] Würz W., Herr S., Wörner A., Rist U., Wagner S., Kachanov, Y.S., Study of 3D wall roughness acoustic receptivity on an airfoil, in: Proc. IUTAM Symposium on Laminar-Turbulent Transition, 13–17 September, Sedona, USA, 1999 (accepted for publication).